

FIT OF FIRST ORDER THERMOLUMINESCENCE GLOW PEAKS USING THE WEIBULL DISTRIBUTION FUNCTION

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Received March 9 2000, amended June 20 2000, accepted July 3 2000

Abstract — A new thermoluminescence glow curve deconvolution (GCD) function is introduced which accurately describes first order thermoluminescence (TL) curves. The new GCD function is found to be accurate for first order TL peaks with a wide variety of the values of the TL kinetic parameters E and s . The 3-parameter Weibull probability function is used with the function variables being the maximum peak intensity (I_m), the temperature of the maximum peak intensity (T_m) and the Weibull width parameter b . An analytical expression is derived from which the activation energy E can be calculated as a function of T_m and the Weibull width parameter b . The accuracy of the Weibull fit was tested using the ten reference glow curves of the GLOCANIN intercomparison program and the Weibull distribution was found to be highly effective in describing both single and complex TL glow curves. The goodness of fit of the Weibull function is described by the Figure of Merit (FOM) which is found to be of comparable accuracy to the best FOM values of the GLOCANIN program. The FOM values are also comparable to the FOM values obtained using the recently published GCD functions of Kitis *et al.* It is found that the TL kinetic analysis of complex first-order TL glow curves can be performed with high accuracy and speed by using commercially available software packages.

INTRODUCTION

During the past decade the deconvolution of thermoluminescence (TL) glow curves by computerised glow curve analysis has become very popular. Recently Bos *et al.*^(1,2) have compared the accuracy of various methods commonly used to analyse complex thermoluminescence glow curves under the intercomparison program GLOCANIN. Participants in the program were asked to analyse ten Reference Glow Curves (RGC) of LiF(TLD-100), the most popular material in TL dosimetry.

In this paper a new thermoluminescence glow curve deconvolution (GCD) function is introduced, which accurately describes a first order single thermoluminescence (TL) glow peak. The GCD function is found to be accurate for a wide range of the values of the TL kinetic parameters E and s . The 3-parameter Weibull probability function is used here with the function variables being the maximum peak intensity (I_m), the temperature of the maximum peak intensity (T_m) and the Weibull width parameter b . An analytical expression is derived from which the activation energy E can be calculated using T_m and the Weibull width parameter b . The expression accurately reproduces the activation energy E within 1% of the reference values.

The Weibull distribution function described here has several similarities with the TL glow curve deconvolution functions recently published by Kitis *et al.*⁽³⁾. These authors introduced GCD functions that are based on two experimental parameters, namely the maximum peak intensity (I_m) and the corresponding temperature

T_m . The variable parameter in their GCD functions is the activation energy E .

The accuracy of the Weibull fit was tested using the ten reference glow curves of the GLOCANIN intercomparison program. It was found that the Weibull distribution is highly effective in describing both single and complex TL glow curves. The Figure of Merit (FOM) of the Weibull distribution is found to be of comparable accuracy to some of the best FOM values of the GLOCANIN program, as well as to the FOM values obtained using the GCD functions of Kitis *et al.*⁽³⁾.

It is found that TL kinetic analysis of complex first order TL glow curves can be performed with high accuracy and speed by using commercially available software packages.

KINETIC EQUATIONS — THE WEIBULL PROBABILITY FUNCTION

The TL intensity for peaks exhibiting first order kinetics is given by the well known first order kinetics equation⁽⁴⁾:

$$I(T) = s n_0 \exp(-E/kT) \exp\left[-\frac{s}{\beta} \int_{T_0}^T \exp(-E/kT') dT' + 1\right] \quad (1)$$

where β is the heating rate, T_0 is the initial temperature and the parameter s is the usual frequency factor which

has the dimensions of s^{-1} . Here n_0 represents the initial number of filled traps, E is the activation energy for the TL process and k is the Boltzmann constant. Recently Kitis *et al.*⁽³⁾ developed glow curve deconvolution (GCD) functions for first, second and general order kinetics which use the experimentally determined maximum intensity I_m and the corresponding temperature of maximum intensity T_m . The expressions derived for the TL intensity contain I_m, T_m and the kinetic parameters E and b . By comparing the GCD functions with specific synthetic and experimental TL glow curves, Kitis *et al.*⁽³⁾ showed that their GCD functions yield accurate values of the energy E , within 3% of the correct values. Their equation for TL peaks involving first order kinetics is:

$$I(T) = I_m \left\{ 1 + \left(\frac{E}{kT} \frac{T - T_m}{T_m} \right) - \left(1 - \frac{2kT}{E} \right) \left[\frac{T^2}{T_m^2} \exp \left(\frac{E}{kT} \frac{T - T_m}{T_m} \right) - \frac{2kT_m}{E} \right] \right\} \quad (2)$$

Here I_m represents the maximum TL intensity and T_m is the corresponding temperature.

In this paper it is shown that a first order single TL peak is accurately described by the Weibull distribution function and the results are compared to the GCD function in Equation 2 as well as to the numerically integrated Equation 1. The Weibull probability distribution function is commonly used for statistical analysis because of its ability to provide a good fit for a variety of data sets^(5,6). It is commonly used in quality process control and has been used to describe such diverse physical phenomena as the time of failure of electronic components, the time of wear of automobile tyres, and the distribution of winds over a year. The two-parameter Weibull function is also found as a standard component of commercially available spreadsheets such as EXCEL.

Mathematically the general 3-parameter Weibull distribution is given by the equation⁽⁷⁾

$$W(T) = I_m \left(\frac{c-1}{c} \right)^{\frac{1-c}{c}} g(T)^{c-1} \exp \left[-g(T)^c + \frac{c-1}{c} \right] \quad (3)$$

where

$$g(T) = \frac{T - T_m}{b} + \left(\frac{c-1}{c} \right)^{\frac{1}{c}} \quad (4)$$

In the case of TL glow curves the parameter T represents the temperature in degrees K, while T_m represents the centre of the Weibull distribution. The parameter b is known as the scale or width of the Weibull function and c is called the asymmetry parameter of the distribution. The parameter I_m represents the maximum height of $W(T)$ at the centre point $T = T_m$. For first order TL peaks it is found by a least squares fitting procedure that the value of $c = 16$ gives the best fit for all first order TL peaks, and therefore the value of $c = 16$ is

fixed throughout the present work. By using $c = 16$ the Weibull equation above becomes

$$W(T) = 2.713 I_m \left(\frac{T - T_m}{b} + 0.996 \right)^{15} \exp \left(- \left(\frac{T - T_m}{b} + 0.996 \right)^{16} \right) \quad (5)$$

An example of comparing the numerically integrated Equation 1 with the GCD functions 2 and with the Weibull Equation 5 is shown in Figure 1 for specific values of the energy E and the frequency s . Several other examples are shown in Table 1 for a wide range of values of E and s . The results in Figure 1 show that the Weibull function with $c = 16$ accurately describes first order TL glow peaks assuming linear heating, i.e. the temperature increases linearly with time. The goodness

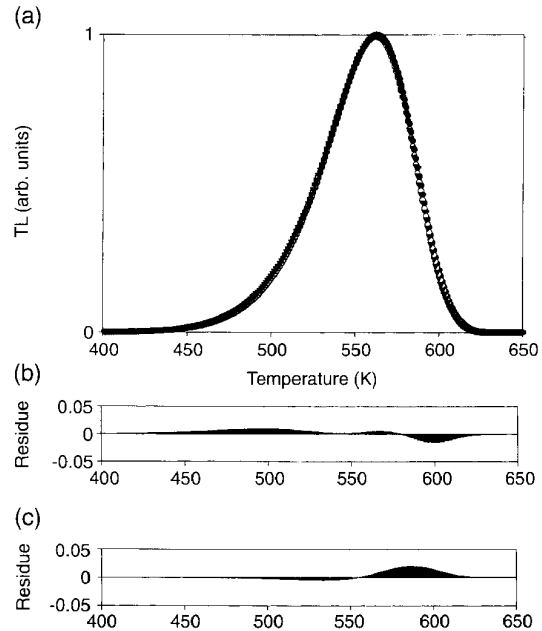


Figure 1. Comparison of the TL glow curve obtained by numerical integration of Equation 1 with the Weibull function with parameters $T_m = 562$ K, $b = 403.7$ K and $I_m = 1$. The parameters used in Equation 1 are $E = 1.0$ eV and $s = 3.10^7 s^{-1}$ and a heating rate of $\beta = 1^\circ C.s^{-1}$. The recently published GCD Equation 2 is also shown for comparison with the parameters $T_m = 562$ K, $E = 1.0$ eV and $I_m = 1$ (arbitrary units). All 3 curves are normalised to yield a normalised value of maximum intensity equal to 1. The Figure of Merit using the Weibull function 5 is of the order of 10^{-5} , of the same order as the one obtained using the GCD function 2. Similar good fits were obtained for a wide variety of the values of E , s as shown in Table 1. Figure 1(b) shows the deviations between Equations 1 and 5. Figure 1(c) shows the deviations between Equations 1 and 2.

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of fit is expressed mathematically by the Figure of Merit FOM⁽²⁾ which is defined by:

$$FOM = \sum_p \frac{|y_{\text{experimental}} - y_{\text{fit}}|}{\text{AREA}_{\text{fit}}}$$

where $y_{\text{experimental}}$ and y_{fit} represent the experimental data and the values of the fitting function correspondingly. The summation extends over all the available points and Area_{fit} represents the integral of the fitted glow curve. In the example of Figure 1, very low values of $FOM = 2.6 \times 10^{-5}$ are obtained using the Weibull Equation 5, while a value of $FOM = 3.2 \times 10^{-5}$ is obtained using the GCD function 2. Very good values of the FOM of the order of 10^{-3} or better are obtained for a wide variety of the values of the energy E and the frequency factor s as shown in Table 1.

The success of the Weibull function in describing the first order TL peaks is explained in the next section, where Equations 2 and 5 are compared in more detail and the mathematical properties of the Weibull are studied. The accuracy of the Weibull fits is also found to be comparable with the methods of the GLOCANIN intercomparison program. These results are presented in the next to last section of the paper.

MATHEMATICAL PROPERTIES OF THE WEIBULL FUNCTION

In this section the mathematical behaviour of the Weibull function is studied for various values of the temperature T. Specifically it is shown that at low values of the temperature T the Weibull function correctly approximates the well known initial rise expression $e^{-E/kT}$ for TL glow curves. The behaviour of the Weibull

is also examined near the temperature of maximum intensity $T = T_m$ and an analytical relationship is derived between the activation energy E and the parameters b and T_m . Finally an analytical linear relationship between the Weibull width parameter b and the full-width at half-maximum (FWHM) of the TL glow curve is derived, and the shape factor $\mu = \delta/\omega$ of the Weibull distribution is shown to be equal to that of a first order TL glow peak.

At low temperatures T the exponential factor in the Weibull expression 5 approaches the value of 1 and Equation 5 becomes

$$W(T) = 2.713 I_m \left(\frac{T - T_m}{b} + 0.996 \right)^{15} \quad (6)$$

This equation is compared in Figure 2(a) with the well-known initial rise expression $e^{-E/kT}$, where E is the activation energy and k is the Boltzmann factor. In conclusion, the Weibull function accurately describes the behaviour of first order TL peaks at low temperatures. Next the behaviour near the temperature of maximum intensity is studied in detail.

The relationship between the activation energy E and the Weibull parameters T_m and b can be found by a Taylor expansion of Equation 5 near the temperature of maximum intensity T_m . The first three terms of the Taylor expansion of Equation 5 are easily found to be given by:

$$W1(T) = I_m \left(1 - \frac{0.006178}{b} (T - T_m) - \frac{121.018}{b^2} (T - T_m)^2 + O(T^3) \right) \quad (7)$$

Table 1. Accuracy of the Weibull analysis for first order TL glow curves.

Weibull parameters		Calculated activation energy E_{Weibull} (eV)	Actual activation energy E_{actual} (eV)	Weibull % accuracy	Frequency factor s, (s ⁻¹)	Heating rate β (°C.s ⁻¹)	FOM	Max temperature from Weibull T_{max} (W) (K)	Max temperature from E,s T_{max} (actual) (K)	ΔT (K)
562	403.7	1.0145	1.00	1.45	3×10^7	1	0.0014	562	561.9	-0.1
466	280.9	1.0079	1.00	0.79	3×10^9	1	0.0057	466	465.8	-0.2
347	157.0	1.0020	1.00	0.20	3×10^{13}	1	0.0042	347	345.6	-1.4
290	110.5	0.9965	1.00	-0.35	3×10^{16}	1	0.0095	290	288.9	-0.1
263	128.0	0.7034	0.70	0.49	3×10^{12}	1	0.000004	263	261.5	-1.5
335	162.1	0.9026	0.90	0.29	3×10^{12}	1	0.0026	335	333.7	-1.3
442	212.5	1.2005	1.20	0.04	3×10^{12}	1	0.0029	442	441.1	-0.9
548	262.2	1.5026	1.50	0.17	3×10^{12}	1	0.0028	548	547.7	-0.3
724	343.8	2.0007	2.00	0.04	3×10^{12}	1	0.0031	724	724.1	0.1
377	186.6	0.9984	1.00	-0.16	3×10^{12}	2	0.002	377	377.5	0.5
386	194.5	1.0037	1.00	0.37	3×10^{12}	4	0.0013	386	385.7	-0.3
362	171.9	1.0003	1.00	0.03	3×10^{12}	0.5	0.0032	362	362.1	0.1
345	253.3	0.6088	0.60	1.47	3×10^7	1	0.013	345	344.9	-0.1

Next a Taylor series expansion of the GCD function 2 is performed around $T = T_m$. The first three terms of this Taylor series are found to be:

$$W_2(T) = I_m \left(1 + \frac{6k}{E} (T - T_m) + \frac{-2E^3 k T_m + 6E^2 k^2 T_m^2 - E^4 + 12k^3 T_m^3 E + 36k^4 T_m^4}{2k^2 T_m^4 E^2} (T - T_m)^2 \right) + O(T^3) \quad (8)$$

Figure 2(b) shows the first three terms of the Taylor series 7 and 8 plotted together with the numerically integrated function 1. At temperatures near the temperature of maximum intensity T_m , the first order TL glow peak is accurately described by the first three terms in *both* Taylor expansions. The linear terms in temperature can easily be shown to be negligible near $T = T_m$. By equat-

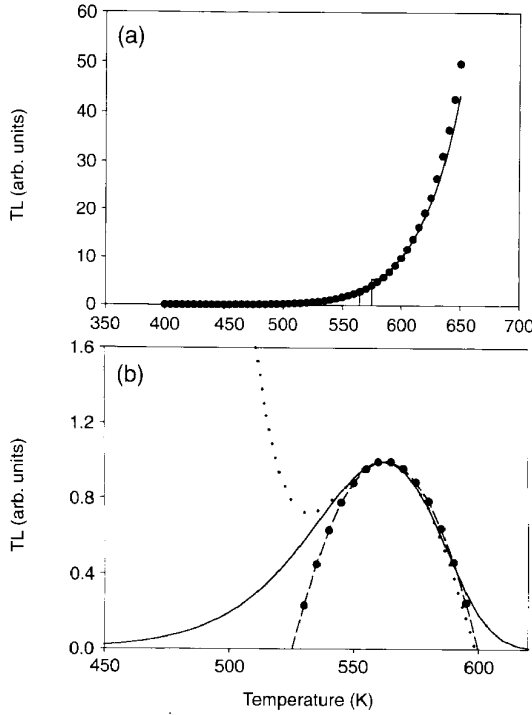


Figure 2. (a) At low temperatures the Weibull function is proportional to the well-known exponential factor $e^{-E/kT}$ used in the initial rise method of TL analysis. (b) At temperatures near the temperature of maximum intensity T_m , the Weibull function is described accurately by the first three terms in the Taylor expansion in Equation 7. It is also described accurately by the first three terms in the Taylor expansion of Equation 8 derived by Kitis *et al.*⁽³⁾. By equating the quadratic terms of the two Taylor series, an analytical expression is derived for the energy E as a function of the Weibull width b and of the temperature of maximum intensity T_m , as described in the text.

ing the quadratic terms of the two Taylor series 7 and 8 an analytical expression is obtained for the energy E as a function of the Weibull width b and of the temperature of maximum intensity T_m .

$$\frac{121.018}{b^2} = \frac{-2 E^3 k T_m + 6E^2 k^2 T_m^2 - E^4 + 12 k^3 T_m^3 E + 36 k^4 T_m^4}{2 k^2 T_m^4 E^2} \quad (9)$$

It can be easily shown that the terms containing k^3 and k^4 are negligible compared with the rest of the terms in the above equation, yielding a quadratic equation in the activation energy E :

$$\frac{121.018}{b^2} = \frac{-2 E^3 k T_m + 6 E^2 k^2 T_m^2 - E^4}{2 k^2 T_m^4 E^2} \quad (10)$$

The solution of this quadratic yields the activation energy E as a function of the Weibull parameters b and T_m :

$$E = T_m \frac{k}{b} [-b + \sqrt{(7 b^2 + 242.036 T_m^2)}] \quad (11)$$

where k is the Boltzmann constant and the Weibull constants T_m and b are given in degrees Kelvin. The results of using Equation 11 to evaluate the activation energy E for a wide variety of the parameters E and s are shown in Table 1. The values of E calculated using Equation 11 are seen to be very accurate, within 1–5% of the actual E values. The last three columns of Table 1 compare the values of Weibull parameter T_m with the corresponding temperature of maximum intensity calculated from the given values of E , s and β . It can be seen that the differences are between 0.1 and 1.5 K, within the accuracy of typical experimental data.

In this section a relationship is derived between the Weibull parameter b and its full-width at half-maximum (FWHM). The low temperature $T_1 < T_m$ and the high temperature $T_2 > T_m$ for which the height of the Weibull is equal to the half-maximum intensity $I_m/2$ are calculated as follows. By equating Equation 5 with $I_m/2$ the equation below is obtained:

$$0.5 I_m = 2.713 I_m \left(\frac{T - T_m}{b} + 0.996 \right)^{15} \exp \left[- \left(\frac{T - T_m}{b} + 0.996 \right)^{16} \right] \quad (12)$$

This equation can be solved numerically and yields the two values

$$\frac{T_1 - T_m}{b} = -0.09034 \quad \frac{T_2 - T_m}{b} = 0.06502 \quad (13)$$

It is concluded that the FWHM of the Weibull is equal to

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$$FWHM = \omega = T_2 - T_1 = (0.06502 + 0.09034)$$

$$b = 0.1553 \omega \text{ or } b = 6.4368 \omega \quad (14)$$

The shape factor for the Weibull distribution is then equal to

$$\mu = \frac{\delta}{\omega} = \frac{0.06502}{0.15536} = 0.418 \quad (15)$$

Hence it is shown that the Weibull distribution has the correct shape factor for a first order TL glow peak⁽⁴⁾. By combining now Equations 11 and 14 an equation is derived which gives the activation energy E as a function of the temperature T_m and the FWHM ω of the Weibull distribution. By inserting the value of b = 6.4368 ω from Equation 14 into Equation 11 it is found after some simple algebra that:

$$(E + k T_m)^2 = 7(k T_m^2) + 242.036 k^2 \frac{T_m^4}{(6.4368 \omega)^2} \quad (16)$$

The term 7(k T_m)² is easily shown to be negligible compared with the second term, so it can be omitted and the above equation becomes:

$$E = \frac{2.417 k T_m^2}{\omega} - k T_m \quad (17)$$

It is noted that Equation 17 has an algebraic similarity with the commonly used first order approximation derived by Chen [Reference 4, p. 114]:

$$E = \frac{2.52 k T_m^2}{\omega} - 2k T_m \quad (18)$$

The values of E obtained using Equation 17 are within 1-2% of the values of E obtained using Chen's Equation 18.

WEIBULL ANALYSIS OF COMPLEX TL GLOW CURVES IN THE GLOCANIN INTERCOMPARISON PROGRAM

The use of the Weibull distribution function was

Table 2. Comparison of FOM values obtained using the Weibull distribution function with the best FOM values from the GLOCANIN intercomparison program.

File name	Present FOM	Best FOM	FOM from Reference 3
REFGLOW.001	0.0017	0.00002	0.00006
REFGLOW.002	0.0019	0.0001	0.0001
REFGLOW.003	0.013	0.0101	0.0104
REFGLOW.004	0.011	0.011	0.0106
REFGLOW.005	0.008	0.0102	0.0105
REFGLOW.006	0.011	0.0094	0.0142
REFGLOW.007	0.0067	0.0094	0.0091
REFGLOW.008	0.0062	0.0091	0.0079
REFGLOW.009	0.0098	0.0054	0.0106
REFGLOW.010	0.036	0.0412	0.0342

tested using the reference glow curves of the GLOCANIN intercomparison program^(1,2). This program contains both synthetic and measured glow curves for the most common TL material LiF:Mg, Ti (TLD-100). In Table 2 the FOM values obtained using the Weibull function are compared with the best FOM values within the GLOCANIN intercomparison program, as well as with the FOM values obtained by Kitis *et al.*⁽³⁾ using the GCD function in Equation 2 of this paper. Table 2 shows that the FOM values obtained from fitting the first two synthetic glow curves are very good, although they are poorer by an order of magnitude from the best FOM in GLOCANIN. This is to be expected, since the Weibull distribution is not a kinetic function but rather an empirical expression.

However, the situation is much different in the case of measured (experimental) glow curves, where the FOM values are very close to the best FOM values within the GLOCANIN program. In three of the eight cases of measured glow curves the Weibull gives the

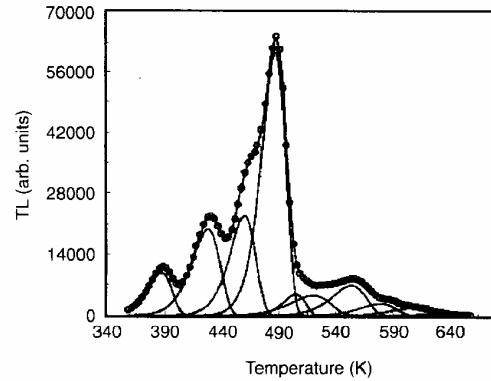
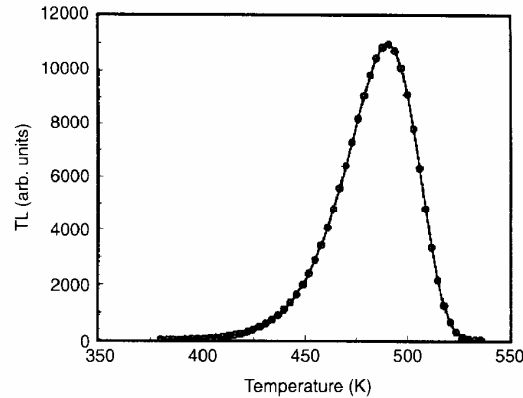


Figure 3. Two examples of the best fits obtained using the Weibull distribution functions to analyse the glow curves in the GLOCANIN intercomparison program. The reference glow curves No 1 and No 9 are shown.

best FOM among all methods. Two examples of the best fits obtained using the Weibull distribution functions are shown in Figure 3.

Table 3 shows a comparison between the activation energy E in the Weibull analysis as evaluated using Equation 11 with the actual activation energy of the GLOCANIN program. The agreement is seen to be very good, with the E values from the Weibull analysis differing by 2.5% or less from the actual E values of GLOCANIN. Similar good agreements are obtained between the calculated and fitted areas as shown in Table 3, with the differences being less than 5.5%. It must be noted that the results for peaks 4 and 5 shown in Table 3 are strongly correlated to each other, due to the strong overlap of these peaks. A positive deviation in the area of peak 4 will be compensated by a negative deviation of peak 5 and vice versa. Table 4 shows a comparison between the Mean Activation Energies of the GLOCANIN report E(GLOC) with the activation energies E obtained using the Weibull functions. The agreement is again seen to be good, with the E values from the Weibull analysis differing typically by 5% or less from the actual E values of GLOCANIN. Only a few of the energy values differ by as much as 9%.

DISCUSSION AND CONCLUSION

Several researchers have attempted previously to use

analytical equations to fit first order TL glow curves. These functions have been summarised in the inter-comparison papers of Bos *et al*^(1,2). Several programs in GLOCANIN use Gaussian functions and modified asymmetric Gaussians to approximate the first order TL glow curves with a varying degree of success. Their results for the activation energy E and/or the areas A under the TL glow curves showed significant deviations from the reference values. The disadvantage of these methods is that they are not based on a physical model of the TL process.

The Weibull function approaches a first order glow peak in a unique way. The reason is that although it is not a kinetic function, it gives fittings that are almost as accurate as the normal kinetics equations. Moreover, from the Weibull function, an expression for E is deduced which is close to that derived from the kinetics models and with an accuracy of 1% in the E values. This is not possible for the other non-kinetic functions used in the GLOCANIN intercomparison program (Gaussians and modified Gaussians).

In this paper first order TL glow curves are fitted using an empirical analytical expression based on the 3-parameter Weibull function. Two of the parameters I_m , T_m are the same as in the recently published GCD functions of Kitis *et al*⁽³⁾, and can be determined experimentally. The third parameter represents the width b of the Weibull distribution and is found to be proportional

Table 3. Results for the synthetic reference glow curves: Activation energy E (eV).

File name	HR (K.s)	Peaks	Peak No.	E (actual)	E (Fitted)	% deviat.	Area (actual)	Area (Fitted)	% dev.
REFGLOW.001	1	1	1	1.1824	1.1863	-0.3	489918	489915	+0.006
REFGLOW.002	8.4	4	2	1.3824	1.3550	2.0	11100	11284	-1.6
			3	1.4833	1.4940	-0.7	16898	16765	+0.8
			4	1.5832	1.5639	1.2	27401	28900	-5.2
			5	2.0038	2.0533	-2.4	47302	45729	+3.4

Table 4. Comparison of the mean activation energies of the GLOCANIN intercomparison E (GLOC.) with the activation energies E obtained using the Weibull functions.

RGC No.	Peak 2			Peak 3			Peak 4			Peak 5		
	E (GLOC.)	E (eV)	Dev %	E (GLOC.)	E (eV)	Dev %	E (GLOC.)	E (eV)	Dev %	E (GLOC.)	E (eV)	Dev %
3	1.259	1.257	0.16	1.331	1.311	1.5	1.615	1.582	2.04	2.117	2.190	-3.44
4	1.240	1.235	0.40	1.331	1.319	0.9	1.627	1.564	3.87	2.084	2.102	-0.86
5	1.482	1.478	0.27	1.544	1.535	0.58	1.540	1.499	2.66	2.154	2.139	0.69
6	1.480	1.478	0.13	1.510	1.468	2.78	1.510	1.536	-1.72	2.168	2.076	4.24
7				2.091	1.910	8.65	2.118	2.170	-2.4	2.085	1.990	4.55
8				2.059	1.932	6.17	2.147	2.170	-1.07	2.072	2.022	2.41
9	1.245	1.297	-4.10	1.298	1.315	-1.3	1.601	1.591	0.62	2.018	2.026	-0.39
10	1.301	1.335	-2.60	1.496	1.429	-4.47	1.555	1.438	7.5	2.082	2.183	-4.85

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to the FWHM of the first order TL glow curve, as given in Equation 14.

In practical terms, the good fits obtained using the Weibull distribution mean that first order glow curves can be analysed accurately and quickly using commercially available packages that incorporate the Weibull function. There are several advantages of using commercially available software packages for analysing first order TL peaks. The Weibull function is available as part of many such packages like the programs SIGMAPLOT and PEAKFIT developed by SPSS⁽⁸⁾. These commercial software packages have well-developed graphical user interfaces, such as pull down menus and click-and-drag initial adjustments of the positions and widths of the TL peaks. The programs automatically provide a complete statistical analysis such as least square errors, absolute and per cent deviations of all data points, area calculations and confidence intervals for the Weibull parameters at any desired level of confidence, F and chi-squared statistics as well as several standard statistical tests. An example of applying the commercial program PEAKFIT to a complex 5-peak TL glow curve is shown in Figure 4.

Typical computer fitting times for single TL glow peaks are 1–2 s for complete adjustment of the Weibull parameters I_m , T_m and b . Typical fitting times for setting up the initial Weibull parameters in the computer program, running the program and obtaining the correct parameters b and T_m for a 5-peak TLD-100 glow curve are 5–10 min. These software packages are WINDOWS based and can run on most PCs. An attempt was made to use the three-parameter Weibull function for second order as well as for general order kinetics. Although the

Weibull produces reasonably good fits of these TL glow curves, it fails to reproduce accurately their behaviour especially at the wings of the TL glow curves. Further work is in progress to fit second and general order kinetics using other distributions similar to the Weibull.

In conclusion, it was shown that using the Weibull distribution with commercially available software packages provides quick and accurate analysis of first order TL glow curves.

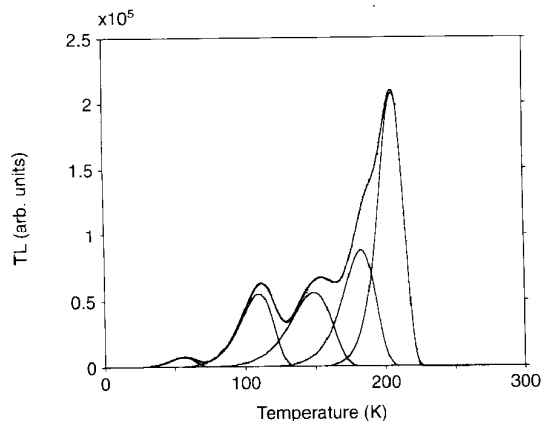


Figure 4. An example of applying the Weibull analysis using the commercially available program PEAKFIT. Here an artificially produced TL glow curve for TLD-100 is analysed using five Weibull functions. The program provides a click and drag type of interface for adjusting the locations and widths of the TL peaks. It also provides a complete set of statistics and detailed analysis of confidence intervals for the parameters of the TL glow curves.

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